Symmetries of mass matrices of family members
Symmetries in all orders of corrections

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21^{st} workshop "What comes beyond the standard models?"

Bled 23 June - 1 July 2018

Some publications:

- arXiv:1112.4368, arXiv:1806.01629, with A. Hernandez-Galeana
- *Phys. Rev.* **D** (**2009**) 80.083534, astro-ph/arXiv: 0907.0196, arXiv:1502.06786, arxiv:1412.5866, **with** G. Bregar,
- *J. of Mod. Phys.* **4** (2013) 823-847, **6** (2015) 2244-2274, arxiv:1409.4981

- **The** *spin-charge-family* **theory predicts the existence of the fourth family to the observed three.**
- **The** 4×4 **mass matrices** — determined by the **nonzero vacuum expectation values** of the **two triplet scalars**, the gauge fields of the two groups of $\widetilde{SU}(2)$ determining **family quantum numbers**, and by the contributions of the **dynamical fields** of these two scalar triplets and the **three scalar singlets** with the **family members quantum numbers** (Q, Q', Y') — manifest the symmetry $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$.
- **All scalars carry the weak and the hyper charge of the** *standard model* **higgs field** ($\pm\frac{1}{2}, \mp\frac{1}{2}$, **respectively**).
- **We are proving, using the massless spinor basis, that the symmetry of the** 4×4 **mass matrices remains** $SU(2) \times SU(2) \times U(1)$ **in all orders of corrections.**

The *spin-charge-family* **theory predicts for the low energy regime:**

- **The existence of the fourth family to the observed three.**
- **The existence of twice two triplets, ($\vec{\tilde{A}}^{\tilde{1}}_{s}$, $\vec{\tilde{A}}^{\tilde{N}_L}_{s}$),** $s = (7,8)$**,** and **three singlets of scalars, ($\vec{A}^{Q}_{s}, \vec{A}^{Q'}_{s}, \vec{A}^{Y'}_{s}$),** $s = (7,8)$, **all with the properties of the higgs with respect to the weak and hyper charges, what explains the origin of the Yukawa couplings.**

The Lagrange density for fermions, coupled only to gravity in $d > 5$ through vielbeins and the two kinds of spin connection fields, the gauge fields of S^{ab} and $\tilde{S}^{ab}$ is

- $$\begin{aligned} \mathcal{L}_f &= \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^{Ai}\tau^{Ai}A^{Ai}_m)\psi + \\ &\{\sum_{\mathbf{s=7,8}} \bar{\psi}\gamma^{\mathbf{s}}\mathbf{p_{0s}}\,\psi\} + \\ &\{\sum_{t=5,6,9,\ldots,14} \bar{\psi}\gamma^{t} p_{0t}\,\psi\} \end{aligned}$$

where $p_{0s} = p_s - \frac{1}{2}S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$,
$p_{0t} = p_t - \frac{1}{2}S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}$,
with $m \in (0,1,2,3)$, $s \in (7,8)$, $(s', s") \in (5,6,7,8)$, (a,b) **(appearing in $\tilde{S}^{ab}$) run within either**
$(0,1,2,3)$ **or** $(5,6,7,8)$, t runs $\in (5,\ldots,14)$, $(t', t")$ run either $\in (5,6,7,8)$ or $\in (9,10,\ldots,14)$.

- **The Lagrange density for the mass term of fermions, coupled to scalar fields origining in gravity in** $d \geq 13+1$,

if expressing $\gamma^7 = (\overset{78}{(+)} + \overset{78}{(-)})$ and correspondingly $\gamma^8 = -i(\overset{78}{(+)} - \overset{78}{(-)})$

$$
\begin{aligned}
\mathcal{L}_{mass} &= \frac{1}{2} \sum_{+,-} \{\psi^\dagger_L \gamma^0 \; \overset{78}{(\pm)} \; (-\sum_A \tau^A A^A_\pm - \sum_{\tilde{A}i} \tilde{\tau}^{Ai} \tilde{A}^{\tilde{A}i}_\pm)\psi_R\} + h \\
\tau^A &= (Q, Q', Y')\,, \quad \tilde{\tau}^{\tilde{A}i} = (\vec{\tilde{N}}_L, \vec{\tilde{\tau}}^{\tilde{1}})\,, \\
\gamma^0 \; \overset{78}{(\pm)} &= \gamma^0 \frac{1}{2}(\gamma^7 \pm i\,\gamma^8)\,, \\
A^A_\pm &= \sum_{st} c^A{}_{st}\, \omega^{st}{}_\pm\,, \quad \omega^{st}{}_\pm = \omega^{st}{}_7 \mp i\,\omega^{st}{}_8\,, \\
\vec{\tilde{A}}^{\tilde{A}}_\pm &= \sum_{ab} c^A{}_{ab}\, \tilde{\omega}^{ab}{}_\pm\,, \quad \tilde{\omega}^{ab}{}_\pm = \tilde{\omega}^{ab}{}_7 \mp i\,\tilde{\omega}^{ab}{}_8\,.
\end{aligned}
$$

The term p_s is left out since at low energies its contribution is negligible.

The contribution of the scalar gauge fields to masses of different family members strongly depends on the quantum numbers Q, Q' **and** Y'. In loop corrections the contribution of the scalar gauge fields of (Q, Q', Y') is proportional to the even power of these quantum numbers.

R	$Q_{L,R}$	Y	$\tau^{4}_{L,R}$	τ^{23}	Y'	Q'	L	Y	τ^{13}	Y'
u^{i}_{R}	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}\,(1-\frac{1}{3}\tan^{2}\vartheta_{2})$	$-\frac{2}{3}\tan^{2}\vartheta_{1}$	u^{i}_{L}	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}\tan^{2}\vartheta_{2}$
d^{i}_{R}	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}\,(1+\frac{1}{3}\tan^{2}\vartheta_{2})$	$\frac{1}{3}\tan^{2}\vartheta_{1}$	d^{i}_{L}	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}\tan^{2}\vartheta^{2}$
ν^{i}_{R}	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}\,(1+\tan^{2}\vartheta_{2})$	0	ν^{i}_{L}	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}\tan^{2}\vartheta_{2}$
e_{R}	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}\,(-1+\tan^{2}\vartheta_{2})$	$\tan^{2}\vartheta_{1}$	e_{L}	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}\tan^{2}\vartheta_{2}$

Table: Quantum numbers $Q, Y, \tau^{4}, Y', Q', \tau^{23}, \tau^{13}$, of the family members $u^{i}_{L,R}, \nu^{i}_{L,R}$ of one family (any one). The left and right handed members of any family have the same Q and τ^{4}, the right handed members have $\tau^{13}=0$, and $\tau^{23}=\frac{1}{2}$ for (u^{i}_{R}, ν^{i}_{R}) and $-\frac{1}{2}$ for (d^{i}_{R}, e^{i}_{R}), while the left handed members have $\tau^{23}=0$ and $\tau^{13}=\frac{1}{2}$ for (u^{i}_{L}, ν^{i}_{L}) and $-\frac{1}{2}$ for (d^{i}_{L}, e^{i}_{L}). ν^{i}_{R} couples only to $A^{Y'}_{s}$ as seen from the table.

- We use the massless basis $\frac{1}{\sqrt{2}}|u^i_L + u^i_R>$.
- Let $\hat{O}$ presents the operators, which determine the mass matrices of family members α — quarks and leptons:

$$\begin{aligned}
\hat{\mathbf{O}} &= \sum_{+,-} \gamma^{\mathbf{0}} \stackrel{\mathbf{78}}{(\pm)} \left(-\sum_{\mathbf{A}} \tau^{\mathbf{A}}\, \mathbf{A}^{\mathbf{A}}_{\pm} - \sum_{\tilde{\mathbf{A}}\mathbf{i}} \tilde{\tau}^{\tilde{\mathbf{A}}\mathbf{i}}\, \tilde{\mathbf{A}}^{\tilde{\mathbf{A}}\mathbf{i}}_{\pm}\right), \\
\tau^{\mathbf{A}}\, \mathbf{A}^{\mathbf{A}}_{\pm} &= (\mathbf{Q}\, \mathbf{A}^{\mathbf{Q}}_{\pm}, \mathbf{Q}'\, \mathbf{A}^{\mathbf{Q}'}_{\pm}, \mathbf{Y}'\, \mathbf{A}^{\mathbf{Y}'}_{\pm}), \\
\tilde{\tau}^{\tilde{\mathbf{A}}\mathbf{i}}\, \tilde{\mathbf{A}}^{\tilde{\mathbf{A}}\mathbf{i}}_{\pm} &= (\tilde{\tau}^{\tilde{\mathbf{1}}\mathbf{i}}\, \tilde{\mathbf{A}}^{\tilde{\mathbf{1}}\mathbf{i}}_{\pm}, \tilde{\mathbf{N}}^{\mathbf{i}}_{\mathbf{L}}\, \tilde{\mathbf{A}}^{\tilde{\mathbf{N}}_{\mathbf{L}}\mathbf{i}}_{\pm}), \\
\{\tau^{\mathbf{A}}, \tau^{\mathbf{B}}\}_- &= \mathbf{0}, \quad \{\tilde{\tau}^{\tilde{\mathbf{A}}\mathbf{i}}, \tilde{\tau}^{\tilde{\mathbf{B}}\mathbf{j}}\}_- = \mathbf{i}\, \delta^{\tilde{\mathbf{A}}\tilde{\mathbf{B}}}\, \mathbf{f}^{\mathbf{ijk}}\, \tilde{\tau}^{\tilde{\mathbf{A}}\mathbf{k}}, \\
\{\tau^{\mathbf{A}}, \tilde{\tau}^{\tilde{\mathbf{B}}\mathbf{j}}\}_- &= \mathbf{0}.
\end{aligned}$$

- **Each of the fields consists in general of the nonzero vacuum expectation value and the dynamical part**: $\tilde{A}^{\tilde{A}i}_{\pm} = (< \tilde{A}^{\tilde{1}i}_{\pm} > + \tilde{A}^{\tilde{1}i}_{\pm}(x)$**,** $< \tilde{A}^{\tilde{N}_L i}_{\pm} > + \tilde{A}^{\tilde{N}_L i}_{\pm}(x)$**,** $< A^{\alpha}_{\pm} > + A^{\alpha}_{\pm}(x)$**)**, **where a common notation for all three singlets** is used, since their eigenvalues depend only on the **family members** $(\alpha = (u, d, \nu, e))$ **and are the same for all the families.**

- Let $\hat{O}^{\alpha} = \sum_{+,-} \gamma^0 \overset{78}{(\pm)} (\tau^4 A^4_{\overset{78}{(\pm)}} + \tau^{23} A^{23}_{\overset{78}{(\pm)}} + \tau^{13} A^{13}_{\overset{78}{(\pm)}})$. One obtains $\frac{1}{\sqrt{2}}\{\frac{1}{6}(A^4_- + A^4_+) + A^{23}_- + A^{13}_+\}$. Equivalent evaluations for $|d^i_L + d^i_R >$ would give $\frac{1}{\sqrt{2}}\{\frac{1}{6}(A^4_- + A^4_+) - A^{23}_- - A^{13}_+\}$, while for neutrinos we would obtain $\frac{1}{\sqrt{2}}\{-\frac{1}{2}(A^4_- + A^4_+) + A^{23}_- + A^{13}_+\}$ and for e^i we would obtain $\frac{1}{\sqrt{2}}\{-\frac{1}{2}(A^4_- + A^4_+) - A^{23}_- - A^{13}_+\}$. Let us point out that the fields include also coupling constants, which change when the symmetry is broken.
- We find

$$
\begin{aligned}
\{\gamma^0 \overset{78}{(\pm)}, \tau^4\}_- &= 0\,, \quad \{\gamma^0 \overset{78}{(\pm)}, \vec{\tilde{\tau}}^{\tilde{1}}\}_- = 0\,, \quad \{\gamma^0 \overset{78}{(\pm)}, \vec{\tilde{N}}_L\}_- = 0\,,\\
\{\gamma^0 \overset{78}{(\pm)}, \tau^{13}\}_- &= -2\,\gamma^0 \overset{78}{(\pm)}\; S^{78}\,,\\
\{\gamma^0 \overset{78}{(\pm)}, \tau^{23}\}_- &= +2\,\gamma^0 \overset{78}{(\pm)}\; S^{78}\,.
\end{aligned}
$$

- $$\hat{U}\,|\psi_R^{\alpha\,i}> \;=\; i\sum_{n=0}^{\infty}\frac{(-1)^n\,\hat{O}^{2n+1}}{(2n+1)!}\,|\psi_R^{\alpha\,i}> \;.$$

- $$\begin{aligned}<\psi_L^{\alpha\,j}|\,\hat{U}\,|\psi_R^{\alpha\,i}> \;=\; & i\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\;<\psi_L^{\alpha\,i}|\sum_{k_1=1}^{4}\hat{O}|\psi_R^{\alpha\,k_1}>\cdot \\ & <\psi_R^{\alpha\,k_1}|\sum_{k_2=1}^{4}\hat{O}|\psi_L^{\alpha\,k_2}>\cdots \\ & <\psi_L^{\alpha\,k_n}|\sum_{k_i=1}^{4}\hat{O}|\psi_R^{\alpha\,k_i}> \;.\end{aligned}$$

- **Our purpose is to show that the matrix elements in all orders of corrections keep the symmetry of $\widetilde{SU}(2)\times\widetilde{SU}(2)\times U(1)$, if $U(1)$ has no nonzero vacuum expectation values.**

- The only operators τ^A, which distinguish among family members, are $(\tau^4, \tau^{13}, \tau^{23})$, included in $Q = (\tau^{13} + Y)$, $Y = (\tau^{23} + \tau^4)$, $Q' = (\tau^{13} - Y \tan^2 \vartheta_1)$ and in $Y' = (\tau^{23} - \tau^4 \tan^2 \vartheta_2)$.
- All the operators contributing to the mass matrices of family members have a factor $\gamma^0 \stackrel{78}{(\pm)}$, which transforms left handed family members to the corresponding right family members and opposite.
- When taking into account $\hat{O}^{2n+1}$ in all orders, the operators $\tau^A A^A_{\pm}$ $(\tau^A = (Q, Q', Y'))$ — $\gamma^0 \stackrel{78}{(\pm)}$ $(\tau^4 A^4_{\pm}, \tau^{23} A^{23}_{\pm}, \tau^{13} A^{13}_{\pm})$ — contribute to all the matrix elements, diagonal and off diagonal ones, **which are nonzero on the tree level.**

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$$
\begin{aligned}
\gamma^0 \stackrel{78}{(-)} |\psi^i_{u_R,\nu_R}> &= |\psi^i_{u_L,\nu_L}>\,,\\
\gamma^0 \stackrel{78}{(+)} |\psi^i_{u_L,\nu_L}> &= |\psi^i_{u_R,\nu_R}>\,,\\
\gamma^0 \stackrel{78}{(+)} |\psi^i_{d_R,e_R}> &= |\psi^i_{d_L,e_L}>\,,\\
\gamma^0 \stackrel{78}{(-)} |\psi^i_{d_L,e_L}> &= |\psi^i_{d_R,e_R}>\,.
\end{aligned}
$$

-
$$
\begin{aligned}
\tilde{N}^{\pm}_L &= -\stackrel{03}{\widetilde{(\mp i)}}\stackrel{12}{\widetilde{(\pm)}}\,, \quad \tilde{\tau}^{1\pm} = (\mp)\stackrel{56}{\widetilde{(\pm)}}\stackrel{78}{\widetilde{(\mp)}}\,,\\
\tilde{N}^{3}_L &= \frac{1}{2}\,(\tilde{S}^{12} + i\,\tilde{S}^{03})\,, \quad \tilde{\tau}^{13} = \frac{1}{2}\,(\tilde{S}^{56} - \tilde{S}^{78})\,,\\
\stackrel{ab}{\widetilde{(-k)}}\stackrel{ab}{(k)} &= -i\,\eta^{aa}\stackrel{ab}{[k]}\,, \quad \stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{(k)} = 0\,,\\
\stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{[k]} &= i\stackrel{ab}{(k)}\,, \quad \stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{[-k]} = 0\,,\\
\stackrel{ab}{\widetilde{(k)}} &= \frac{1}{2}\,(\tilde{\gamma}^a + \frac{\eta^{aa}}{ik}\,\tilde{\gamma}^b)\,, \quad \stackrel{ab}{\widetilde{[k]}} = \frac{1}{2}\,(1 + \frac{i}{k}\,\tilde{\gamma}^a\tilde{\gamma}^b)\,,
\end{aligned}
$$

$$
\begin{aligned}
\tilde{N}_L^{+}\,|\psi^1> &= |\psi^2>\,, \quad \tilde{N}_L^{+}\,|\psi^2>= 0\,,\\
\tilde{N}_L^{-}\,|\psi^2> &= |\psi^1>\,, \quad \tilde{N}_L^{-}\,|\psi^1>= 0\,,\\
\tilde{N}_L^{+}\,|\psi^3> &= |\psi^4>\,, \quad \tilde{N}_L^{+}\,|\psi^4>= 0\,,\\
\tilde{N}_L^{-}\,|\psi^4> &= |\psi^3>\,, \quad \tilde{N}_L^{-}\,|\psi^3>= 0\,,\\
\tilde{\tau}^{1+}\,|\psi^1> &= |\psi^3>\,, \quad \tilde{\tau}^{1+}\,|\psi^3>= 0\,,\\
\tilde{\tau}^{1-}\,|\psi^3> &= |\psi^1>\,, \quad \tilde{\tau}^{1-}\,|\psi^1>= 0\,,\\
\tilde{\tau}^{1-}\,|\psi^4> &= |\psi^2>\,, \quad \tilde{\tau}^{1-}\,|\psi^2>= 0\,,\\
\tilde{\tau}^{1+}\,|\psi^2> &= |\psi^4>\,, \quad \tilde{\tau}^{1+}\,|\psi^4>= 0\,,
\end{aligned}
$$

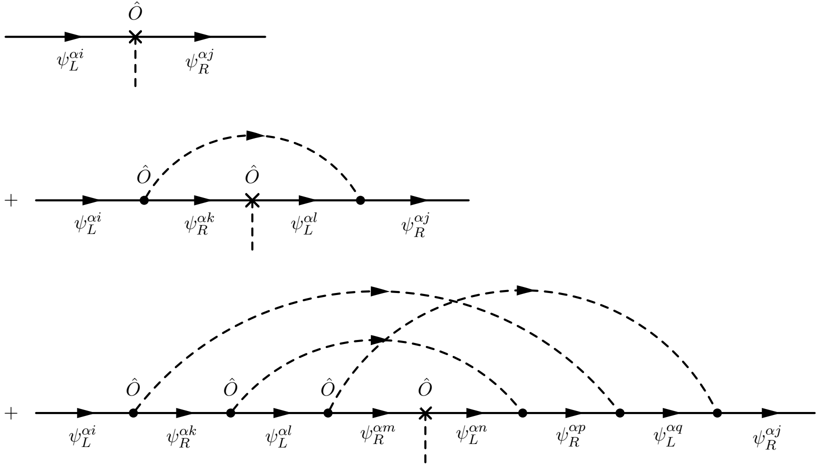
$$
\begin{aligned}
\tilde{N}_L^{3}\,|\psi^1> &= -\frac{1}{2}\,|\psi^1>\,, \quad \tilde{N}_L^{3}\,|\psi^2>= +\frac{1}{2}\,|\psi^2>\,,\\
\tilde{N}_L^{3}\,|\psi^3> &= -\frac{1}{2}\,|\psi^3>\,, \quad \tilde{N}_L^{3}\,|\psi^4>= +\frac{1}{2}\,|\psi^4>\,,\\
\tilde{\tau}^{13}\,|\psi^1> &= -\frac{1}{2}\,|\psi^1>\,, \quad \tilde{\tau}^{13}\,|\psi^2>= -\frac{1}{2}\,|\psi^2>\,,\\
\tilde{\tau}^{13}\,|\psi^3> &= +\frac{1}{2}\,|\psi^3>\,, \quad \tilde{\tau}^{13}\,|\psi^4>= +\frac{1}{2}\,|\psi^4>\,,
\end{aligned}
$$

				$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}_L^3$	$\tilde{N}_R^3$	$\tilde{\tau}$
$\psi^1_{u^{ci}_R}$	$\overset{03}{(+i)}\,\overset{12}{[+]} \mid \overset{56}{[+]}\,\overset{78}{(+)} \mid\mid \cdots$	$\psi^1_{u^{ci}_L}$	$\overset{03}{[-i]}\,\overset{12}{[+]} \mid \overset{56}{[+]}\,\overset{78}{[-]} \mid\mid \cdots$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-$
$\psi^2_{u^{ci}_R}$	$\overset{03}{[+i]}\,\overset{12}{(+)} \mid \overset{56}{[+]}\,\overset{78}{(+)} \mid\mid \cdots$	$\psi^2_{u^{ci}_L}$	$\overset{03}{(-i)}\,\overset{12}{(+)} \mid \overset{56}{[+]}\,\overset{78}{[-]} \mid\mid \cdots$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-$
$\psi^3_{u^{ci}_R}$	$\overset{03}{(+i)}\,\overset{12}{[+]} \mid \overset{56}{(+)}\,\overset{78}{[+]} \mid\mid \cdots$	$\psi^3_{u^{ci}_L}$	$\overset{03}{[-i]}\,\overset{12}{[+]} \mid \overset{56}{(+)}\,\overset{78}{(-)} \mid\mid \cdots$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-$
$\psi^4_{u^{ci}_R}$	$\overset{03}{[+i]}\,\overset{12}{(+)} \mid \overset{56}{(+)}\,\overset{78}{[+]} \mid\mid \cdots$	$\psi^4_{u^{ci}_L}$	$\overset{03}{(-i)}\,\overset{12}{(+)} \mid \overset{56}{(+)}\,\overset{78}{(-)} \mid\mid \cdots$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-$

- Albino's scheme

$$\begin{array}{c} \overset{\tilde{N}_L^{\boxplus}}{\leftrightarrow} \\ \begin{pmatrix} \psi^1 & \psi^2 \\ \psi^3 & \psi^4 \end{pmatrix} \end{array} \updownarrow \tilde{\tau}^{\tilde{1}\boxplus} .$$

- **The diagrams for the tree level, one loop and three loop contributions of the operator $\hat{O}$, determining the masses of quarks and leptons.**
  We allow any order of repeating nonzero vacuum expectation values.



▶

$$
{}^{\alpha}\mathcal{M}_{(o)} =
$$

$$
\begin{pmatrix}
-\tilde{a}_1 - \tilde{a}_2 + a^{\alpha} & < \tilde{A}^{\tilde{N}_L \boxminus} > & < \tilde{A}^{\tilde{1}\boxminus} > & a^{\alpha} < \tilde{A}^{\tilde{1}\boxminus} >< \\
< \tilde{A}^{\tilde{N}_L \boxplus} > & -\tilde{a}_1 + \tilde{a}_2 + a^{\alpha} & a^{\alpha} < \tilde{A}^{\tilde{1}\boxminus} >< \tilde{A}^{\tilde{N}_L \boxplus} > & < \tilde{A}^{\tilde{1}\boxminus} \\
< \tilde{A}^{\tilde{1}\boxplus} > & a^{\alpha} < \tilde{A}^{\tilde{1}\boxplus} >< \tilde{A}^{\tilde{N}_L \boxminus} > & \tilde{a}_1 - \tilde{a}_2 + a^{\alpha} & < \tilde{A}^{\tilde{N}_L \boxminus} \\
a^{\alpha} < \tilde{A}^{\tilde{1}\boxplus} >< \tilde{A}^{\tilde{N}_L \boxplus} > & < \tilde{A}^{\tilde{1}\boxplus} > & < \tilde{A}^{\tilde{N}_L \boxplus} > & \tilde{a}_1 + \tilde{a}_2 -
\end{pmatrix}
$$

- The diagonal terms have on the tree level the symmetry $< \psi^1|..|\psi^1 > + < \psi^4|..|\psi^4 >= 2\, a^{\alpha} = < \psi^2|..|\psi^2 > + < \psi^3|..|\psi^3 >$.
- The off diagonal matrix elements have the properties $< \psi^i|..|\psi^j >^{\dagger} = < \psi^j|..|\psi^i >$:
  $< 1|..|3 >=< 2|..|4 >=< 3|..|1 >^{\dagger} = < 4|..|2 >^{\dagger}$,
  $< 1|..|2 >=< 3|..|4 >=< 2|..|1 >^{\dagger} = < 4|..|3 >^{\dagger}$.
- The off diagonal elements with "three steps needed" the contribution of the fields, which depend on particular family member $\alpha = (u, d, \nu, e)$ enters.
  $< 4|..|1 >^{\dagger} = < 1|..|4 >$ and $< 3|..|2 >^{\dagger} = < 2|..|3 >$.

- **We demonstrate (discuss) conditions under which the symmetry of the matrix elements of the mass matrix remains $\widetilde{SU}(2)\times$ $\widetilde{SU}(2) \times U(1)$ in all orders of corrections — of loop corrections, of repetition of nonzero vacuum expectation values and of both together.**

- $\{\tilde{\tau}^{\tilde{1}i}, \tilde{N}^{j}_{L}\}_{-} = 0\,,$
  $\{\tilde{\tau}^{\tilde{1}i}, \tau^{\alpha}\}_{-} = 0\,,$
  $\{\tau^{\alpha}, \tilde{N}^{j}_{L}\}_{-} = 0\,,$
  τ^{α} represents (Q, Q', Y') (or $\tau^{4}, \tau^{23}, \tau^{13}$).

Let us start with $a^{\alpha} = 0$, which is nonzero vacuum expectation values of all the scalar gauge fields of (Q, Q', Y')

- We start with diagonal terms: $< \psi^i|.....|\psi^i >$.
  On the tree level the symmetry is:
  $\{< \psi^1| < \hat{O}^{\alpha}_{dia} > |\psi^1 > + < \psi^4| < \hat{O}^{\alpha}_{dia} > |\psi^4 >\} - \{< \psi^2| < \hat{O}^{\alpha}_{dia} > |\psi^2 > + \{< \psi^3| < \hat{O}^{\alpha}_{dia} > |\psi^3 >\} = 0$.
- It is easy to see that the tree level symmetry remains in all orders, if only the nonzero vacuum expectation values of $< \tilde{A}^{\tilde{1}3} >= \tilde{a}_1$ and $< \tilde{A}^{\tilde{N}_L 3} >= \tilde{a}_2$ contribute in operators $\gamma^0 \stackrel{78}{(\pm)}\ \tilde{\tau}^{\tilde{1}3} < \tilde{A}^{\tilde{1}3} >$ and $\gamma^0 \stackrel{78}{(\pm)}\ \tilde{N}^3_L < \tilde{A}^{\tilde{N}_L 3} >$.
  At, let say, $2k^{th} + 1$ order of corrections we namely have:
  $\{(-(\tilde{a}_1 + \tilde{a}_2))^{2k^{th}+1} + (\tilde{a}_1 + \tilde{a}_2)^{2k^{th}+1}\} - \{(-(\tilde{a}_1 - \tilde{a}_2))^{2k^{th}+1} + (\tilde{a}_1 - \tilde{a}_2)^{2k^{th}+1}\} = 0$.

- **The contribution of the dynamical terms**, **either** $(QA^{Q},\ Q'A^{Q'},\ Y'A^{Y'})$, **or** $(\tilde{A}^{\tilde{1}3}, \tilde{A}^{\tilde{N}_L 3})$ **do not break the three level symmetry.** **Each of them always appears in an even power, changing the order of corrections by a factor of two or** $2n$ $(|\tau^{A\alpha}A^{A}|^{2n}, |\tilde{A}^{\tilde{1}3}|^{2n}, |\tilde{A}^{\tilde{N}_L 3}|^{2n})$, $\tau^{A\alpha}$ determines $(Q^{\alpha}, Q'^{\alpha}, Y'^{\alpha})$, the summation must be taken over A, adding also $|\tilde{A}^{\tilde{1}3}|^{2n}$ and $|\tilde{A}^{\tilde{N}_L 3}|^{2n}$.
- **There are also other contributions, either those with only nonzero vacuum expectation values or with dynamical fields in addition to nonzero expectation values, in which** $\hat{\tilde{O}}^{\tilde{1}\boxplus}$ **and** $\hat{\tilde{O}}^{\tilde{N}_L\boxplus}$ **together with all kinds of diagonal terms contribute.**

- **The operators $\hat{\tilde{O}}^{\tilde{1}\boxplus}$ transforms ψ^1 into ψ^3 and ψ^2 into ψ^4, while $\hat{\tilde{O}}^{\tilde{N}_L\boxplus}$ transforms ψ^1 into ψ^2 and ψ^3 into ψ^4. Correspondingly the states ψ^1 and ψ^4 take under the application of $\hat{\tilde{O}}^{\tilde{N}_L\boxplus}$ the role of ψ^2 and ψ^3, while ψ^2 and ψ^3 take the role of ψ^1 and ψ^4, carrying correspondingly a changed eigenvalues of $\tilde{\tau}^{\tilde{1}3}$ and $\tilde{N}_L^3$. We easily see that the tree level symmetry remains in all orders.**

- Also for the nondiagonal elements the behaviour of mass matrices is equivalent.
- We conclude that the contribution to $< 1|.....|3 >$ is in any order equal to the contributions to the same order to $< 2|.....|4 >$ and contribution to $< 1|.....|2 >$ in any order of corrections is equal to the contribution of the same order $< 3|.....|4 >$.
- Then it follows equivalently for $< 1|.....|3 >^{\dagger} = < 3|.....|1 >$ and $< 2|.....|4 >^{\dagger} = < 4|.....|2 > = < 3|.....|1 >$ at each order of correction,
  $< 1|.....|2 >^{\dagger}$ is in any order of corrections equal to $< 3|.....|4 >^{\dagger}$.

- **However, for** $a^\alpha = 0$ **the off diagonal matrix elements** $< 1|.....|4 >=< 4|.....|1 >^\dagger$ **and** $< 2|.....|3 >=< 3|.....|2 >^\dagger$ **remain zero in all the loop corrections.**

- **When assuming that** $a^{\alpha} \neq 0$, **the off diagonal matrix elements** $< 1|.....|4 >=< 4|.....|1 >^{\dagger}$ **and** $< 2|.....|3 >=< 3|.....|2 >^{\dagger}$ **become nonzero either on the tree level as well as in all the loop corrections.**
- **But in this case it is easy to show that the symmetry in diagonal and off diagonal matrix elements of the family members mass matrices do not keep the symmetry** $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$.